STATIONARY SUPERSONIC NONEQUILIBRIUM-PLASMA SOURCE

G. A. Luk'yanov

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Electron recombination associated with the expulsion of a monatomic-gas plasma into a vacuum was examined in [1]. It was assumed that recombination by collision and radiation constitutes the basic elementary process. Under certain conditions, dispersion that involves residual ionization of the gas at infinity proved to be possible.

In this paper, we examine the stationary flow in a supersonic spherical source of a nonequilibrium monatomic-gas plasma, which consists of electrons, singly charged ions, and atoms. Among the important practical applications of such flows are the motion of a gas in a nozzle and stationary expulsion into a vacuum.

1. Initial assumptions. System of equations. We examine the flow after loss of ionization equilibrium and discontinuation of ionization events. Furthermore, we assume that thermal equilibrium between the electrons and heavy particles is absent and that the plasma quasi-neutrality conditions are satisfied.

Under these assumptions the continuity equations are

$$\frac{d}{dr}(n_e u_L r^2) = \left(\frac{dn_e}{dt}\right)_p r^2 , \tag{1.1}$$

$$\frac{d}{dr}(n_a u_a r^2) = -\left(\frac{dn_e}{dt}\right)_p r^2 , \tag{1.2}$$

where n is the concentration, u is the velocity, r is the distance from the source pole, and $(dn_e/dt)_p$ is the variation in electron concentration due to recombination.

The subscripts e, i, a, L, and m refer to electrons, ions, atoms, charged particles, and heavy particles, respectively. The quasi-neutrality condition $n_e = n_i$, in combination with the electron and ion continuity equations, yields the equality of charged-particle velocities $u_e = u_i = u_L$.

At low temperatures $kT \ll I$ (where I is the ionization potential) and not too small densities, the recombination process is of a cascade nature [1]. First an electron, after triple collision with an ion and another electron, is captured at one of the upper atomic levels. Then, under the influence of electron impacts of the second kind, and subsequently owing to radiation transitions, the bound electron descends onto the ground level. In this case, the change in electron concentration due to recombination is defined by the expression [2]

$$\left(\frac{dn_e}{dt}\right)_p = -aT_e^{-9/2} n_e^3, \qquad a = \frac{e^{10}}{9 m_e^{1/2} k^{9/2} \varepsilon^5 (4\pi)^{7/2}} . \tag{1.3}$$

where e is the electron charge, ε is the dielectric constant in vacuo, m_e is the electron mass, and k is Boltzmann's constant.

Keeping in mind that collisions between heavy particles are responsible for friction between charged and neutral particles, we write the equations of motion in the form

$$\frac{dp_e}{dr} = -en_e E , \tag{1.4}$$

$$m_i n_e u_L \frac{du_L}{dr} = -n_e n_a \varepsilon_{ia} (u_L - u_a) - \frac{dp_i}{dr} + en_e E , \tag{1.5}$$

$$m_a n_a u_a \frac{du_a}{dr} = -n_a n_e \varepsilon_{ai} (u_a - u_L) - \frac{dp_a}{dr} . \tag{1.6}$$

where p is pressure, E is the electric field strength, and $\varepsilon_{ia} = \varepsilon_{ai}$ is the friction coefficient between the atoms and ions.

According to [3], the friction coefficient for solid smooth spheres is defined by the relation

$$\varepsilon_{ia} = \varepsilon_{ai} = \frac{8}{3}\left(\frac{2}{\pi} kT_m \frac{m_a m_i}{m_i + m_a}\right)^{1/2} Q_{ia}, \tag{1.7}$$

where Q_{ia} is the collision cross section between atoms and ions.

In the temperature range $kT \ll I$ under consideration, the ion-atom collision cross section depends rather weakly on the gas temperature [4]. In the following, for simplicity, we assume Q_{ia} = const.

The energy equations for electrons and heavy particles are

$$\frac{d}{dr}\left(n_e u_L r^2 \frac{5}{2} kT_e\right) = Q^*_{em} r^2 - I^* \frac{d}{dr}(n_e u_L r^2) - n_e u_L r^2 eE, \tag{1.8}$$

$$\frac{d}{dr}\left[n_e u_L r^2\left(\frac{5}{2} kT_m + \frac{m_a u_L^2}{2}\right) + n_a u_a r^2\left(\frac{5}{2} kT_m + \frac{m_a u_a^2}{2}\right)\right] = -Q_{em}^* r^2 + n_e u_L r^2 eE. \tag{1.9}$$

where Q_{em}^* is the rate of energy transfer from the electrons to the heavy particles in the case of elastic collisions.

For a degree of ionization above 10^{-3}, energy transfer to the heavy particles occurs primarily due to electron-ion collisions, since the electron-ion collision cross sections exceed by more than 10^3 times the electron-atom cross sections.

For strong ionization $Q_{em}^* = Q^*_{ei}$, and in conformance with [5]

$$Q_{ei}^* = \frac{n_e^2 e^4}{16\pi^2 m_a \varepsilon^2}\left(\frac{8\pi m_e}{kT_e}\right)^{1/2}\left(\frac{T_m}{T_e} - 1\right)\ln\left[\frac{72\pi^2 (k\varepsilon T_e)^3}{n_e e^6}\right]. \tag{1.10}$$

Here, I^* is the energy restored to the electron gas in the deactivation of excited atoms by impacts of the second kind. The other part of the ionization energy $I - I^*$, is irradiated in the spectral lines. Generally, a certain portion of this energy can be converted to heat as a result of possible impact deactivation of an atom excited by resonance radiation. The mean free path of resonance radiation is defined by the density distribution and the nature of spectral line broadening. As far as energy is concerned, the principal role is played by reabsorption of resonance transition from the second to the first level (transition 2-1).

Close to the source pole the plasma is almost always considered optically thick for the resonance transition 2-1, in view of the extremely large effective absorption cross section (between 10^{-9} and 10^{-10} cm^2 at the line center).

For an optically thin plasma, one may use the expression obtained in [1] for the hydrogen atom,

$$I^* = I \times \begin{cases} 4.3 \cdot 10^{-6} n_e^{1/3} T_e^{-1/2} & (kT_e < I^* < I'), \\ 3.1 \cdot 10^{-5} n_e^{1/6} T_e^{1/12} & (I^* > I') \end{cases} \qquad I' = \frac{1}{2} kT_e \left(\frac{2I}{kT_e}\right)^{1/3}. \tag{1.11}$$

In view of the slow variation of I^*, it is sufficient in practice to consider only the second case, $I^* > I'$. If $I^*(r_1) > I'(r_1)$, this inequality is preserved for $r > r_1$. If, on the other hand, $I^*(r_1) < I'(r_1)$, then owing to the relatively slow variation of I^*, the latter inequality changes direction shortly after the cross section $r = r_1$. In the following we use only the second expression in (1.11). For an optically thick plasma, in the case of resonance transition 2-1, the expression I^* takes the form

$$I^* = 3.1 \cdot 10^{-5} n_e^{1/6} T_e^{1/12} + I_{21}. \tag{1.12}$$

For hydrogen (I = 13.53 eV), I_{21} = 10.15 eV and reabsorption of resonance radiation in the transition 2-1 plays a determining role in the transfer of recombination energy to the electron gas.

The equations of state are written in the conventional form

$$p_e = n_e k T_e, \; p_i = n_i k T_m, \quad p_a = n_a k T_m. \tag{1.13}$$

We eliminate n_a, E, p_e, p_m, and p_i from the system of equations and introduce the following dimensionless parameters:

$$x = \frac{r}{r_0}, \quad v_L = \frac{u_L}{u_0}, \quad v_a = \frac{u_a}{u_0}, \quad c_e = \frac{n_e}{n_0},$$

$$\theta_e = \frac{T_e}{T_0}, \quad \theta_m = \frac{T_m}{T_0}, \quad Y = \frac{r_0 e^4 (8\pi m_e)^{1/2} n_0}{40\pi^2 m_a u_0 \varepsilon^2 (kT_0)^{3/2}} \ln\left[\frac{72\pi^2 (kT_0\varepsilon)^3}{n_0 e^6}\right],$$

$$\Pi = \frac{a n_0^2 r_0}{T_0^{9/2} u_0}, \quad \Phi = \frac{8\sqrt{kT_0}\, Q_{ia} r_0 n_0}{3\sqrt{\pi m_a}\, u_0}, \quad W = \frac{kT_0}{m_a u_0^2}, \quad R = \frac{I^*}{{}^3/_2 kT_0}.$$

Here, the subscript 0 denotes heavy-particle parameters which refer to the initial radius r_0 of the supersonic source.

The system of equations for determining v_L, v_a, c_e, θ_e, and θ_m takes the form

$$\frac{d}{dx}(c_e v_L x^2) = -\Pi \frac{c_e^3}{\theta_e^{9/2}} x^2, \tag{1.14}$$

$$c_a v_L \frac{dv_L}{dx} = -\Phi c_e (v_L - v_a)\, \theta_m^{1/2} \left(\frac{1}{x^2 v_a} - \frac{c_e v_L}{v_a}\right) - W \frac{d}{dx}[c_e(\theta_m + \theta_e)], \tag{1.15}$$

$$\left(\frac{1}{x^2 v_a} - \frac{c_e v_L}{v_a}\right) v_a \frac{dv_a}{dx} = -\Phi c_e (v_a - v_L)\, \theta_m^{1/2} \left(\frac{1}{x^2 v_a} - \frac{c_e v_L}{v_a}\right) - W \frac{d}{dx}\left[\theta_m \left(\frac{1}{x^2 v_a} - \frac{c_e v_L}{v_a}\right)\right], \tag{1.16}$$

$$\frac{d}{dx}(c_e v_L x^2 \theta_e) = -Y x^2 c_e^2 \theta_e^{-1/2} \left(1 - \frac{\theta_m}{\theta_e}\right) + R\Pi \frac{c_e^3}{\theta_e^{9/2}} x^2 + \frac{2}{5} v_L x^2 \frac{d}{dx}(c_e\theta_e), \tag{1.17}$$

$$\frac{d}{dx}\left[\left(\theta_m + 0.2\frac{v_a^2}{W}\right) + c_e v_L x^2 0.2 \frac{1}{W}(v_L^2 - v_a^2)\right] = Y x^2 c_e^2 \theta_e^{-1/2}\left(1 - \frac{\theta_m}{\theta_e}\right) - \frac{2}{5} v_L x^2 \frac{d}{dx}(c_e\theta_e). \tag{1.18}$$

2. Flow in the neighborhood of the source pole. In the neighborhood of the source pole, for a sufficiently large density in the initial cross section, where $\Phi \gg 1$, one may set $v_L = v_a = v$. Then system (1.14)–(1.18) takes the form

$$\frac{d}{dx}(c_e v x^2) = -\Pi \frac{c_e^3}{\theta_e^{9/2}} x^2, \tag{2.1}$$

$$c_e v \frac{dv}{dx} = -W \frac{d}{dx}[c_e(\theta_m + \theta_e)], \tag{2.2}$$

$$\frac{d}{dx}(c_e v x^2 \theta_e) = -Y x^2 c_e^2 \theta_e^{-1/2}\left(1 - \frac{\theta_m}{\theta_e}\right) + R\Pi \frac{c_e^3}{\theta_e^{9/2}} x^2 + \frac{2}{5} v x^2 \frac{d}{dx}(c_e\theta_e), \tag{2.3}$$

$$\frac{d}{dx}\left(\theta_m + 0.2\frac{v^2}{W}\right) = Y x^2 c_e^2 \theta_e^{-1/2}\left(1 - \frac{\theta_m}{\theta_e}\right) - \frac{2}{5} v x^2 \frac{d}{dx}(c_e\theta_e). \tag{2.4}$$

System (2.1)–(2.4) was numerically integrated on an Ural-1 computer. Figure 1 shows the results of the computations for an optically thin plasma (1.11) and the initial parameters $r_0 = 10^{-2}$ m, $n_0 = 10^{23}$ m^{-3}, $T_0 = 5 \cdot 10^3$ °K, $u_0 = 10^4$ m/sec, and $m_a = 1.67 \cdot 10^{27}$ kg.

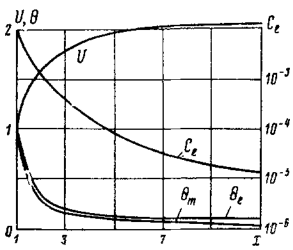

Fig. 1

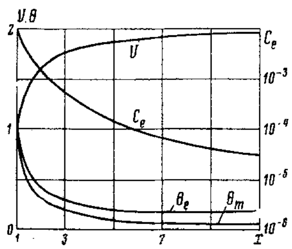

Fig. 2

Similar calculations for an optically thick plasma (1.12), with the same initial conditions, show an appreciable breakaway of the electron temperature (Fig. 2) and a substantial freezing of the degree of ionization α (Fig. 3). Curves 1 and 2 in Fig. 3 refer, respectively, to a plasma optically thin and optically thick with respect to resonance radiation.

This can be interpreted by the relatively large value of the energy (I_{21} = 10.15 eV) which is transferred to the electron gas as a result of impact deactivation of an atom excited by resonance radiation.

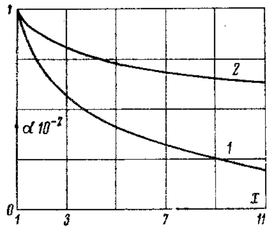

Fig. 3

Calculation results show that for the given initial conditions, conversion of thermal to kinetic energy proceeds with a high intensity and practically terminates in the region $x \lesssim 10$.

Of interest is the change in plasma velocity in the transition from an optically thin to an optically thick plasma. In the second case, in the space region in which the calculations were performed, the velocity proved to be smaller in spite of the fact that the maximum velocity achieved in the total conversion of enthalpy to kinetic energy is larger in the case of an optically thick plasma. In spite of the large value of the recombination energy I* which is transferred to the electrons, the decrease in the recombination rate in an optically thick plasma, owing to the higher electron temperature, leads to a reduction of the velocity gradient and an extension of the zone of thermal to kinetic energy conversion. The nature of the change in the degree of ionization is an indication of the tendency of the flow to freeze its content. In the region of the source where $v \rightarrow$ const and $\alpha \rightarrow$ const, it may be roughly assumed that the electron concentration is $n_e \sim r^{-2}$.

For $R \gg 1$ and $Y \gg 1$ near the initial surface, the electron energy balance is essentially defined by the losses due to elastic collisions with heavy particles and by the restoration to the electron gas of the recombination energy I* which, on the basis of Eq. (1.12), may be roughly taken as a constant,

$$Yx^2c_e^2\theta_e^{-1/2}\left(1-\frac{\theta_m}{\theta_e}\right) = R\Pi c_e^3\theta_e^{-9/2}x^2 \quad \text{or} \quad c_e = \frac{YI}{R\Pi}\theta_e^3(\theta_e-\theta_m)\,. \tag{2.5}$$

Setting $v_L = v_a = v$, we add Eqs. (1.17) and (1.18). Then, using Eq. (1.14) we obtain

$$(R+\theta_e)\,c_e v x^2 + \theta_m + \frac{0.2}{W}v^2 = \frac{A_0}{\Pi}\,, \tag{2.6}$$

where A_0 is a constant determined from the conditions at the initial surface.

From (2.5) and (2.6) in the initial expansion phase, we obtain an expression for the degree of thermal nonequilibrium

$$\frac{\theta_e-\theta_m}{\theta_e} = \frac{R}{Yvx^2\theta_e^4(R+\theta_e)}\left[A_0-\left(0.2\,\frac{\Pi}{W}\,v^2+\Pi\theta_m\right)\right]. \tag{2.7}$$

For $R \gg \theta_e$ ($R > 1$ for an optically thin plasma and $R \gg 1$ for an optically thick plasma), which usually occurs starting with x greater than about 3-5, expression (2.7) simplifies somewhat,

$$\frac{\theta_e-\theta_m}{\theta_e} = \frac{A_0-(0.2\Pi v^2/W+\Pi\theta_m)}{Yvx^2\theta_e^4}\,. \tag{2.8}$$

From (2.8) it follows that the degree of thermal nonequilibrium increases with decreasing initial concentration and increasing atomic mass. Figure 4 shows the results of numerical calculations of the variation of electron and heavy-particle temperatures in a supersonic flow, for hydrogen and argon. The initial concentrations, temperatures, and degrees of ionization were taken in both cases as $n_0 = 10^{23}\ m^{-3}$, $T_0 = 5 \cdot 10^3$ °K, and $\alpha \cdot 10^{-2}$. The plasma was assumed to be optically thick for the resonance transition 2-1. A pronounced difference in the electron temperatures is observed when the heavy-particle temperatures are very similar.

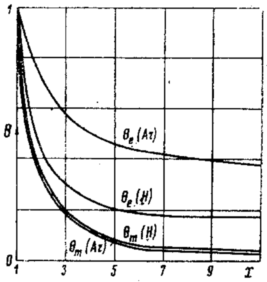

Fig. 4

3. **Flow at a large distance from the source pole.** The electron and ion diffusion rates with respect to neutral atoms can be obtained from the corresponding equations of motion (1.15) and (1.16). For the sake of simplicity, we limit the analysis to the case in which, owing to impairment of thermal equilibrium, the electron temperature is much greater than that of the heavy particles, $\theta_e \gg \theta_m$. When the degree of ionization is pronounced, the pressure in the plasma is defined by the electron pressure. Keeping this in mind, we write the equation for the rate of ambipolar diffusion as

$$\frac{d(v_L - v_a)}{dx} = -\Phi \frac{\theta_m^{1/2}}{x^2 v_a v_L}(v_L - v_a) - \frac{W}{c_e v_L}\frac{d}{dx}(c_e\theta_e). \tag{3.1}$$

When the friction between heavy particles is high, the difference between the terms in the right-hand side of Eq. (2.8) is small compared with each of the terms individually. In this case, the diffusion rate is defined by the expression

$$v_L - v_a = -\frac{W x^2 v_a}{c_e \theta_m^{1/2}\Phi}\frac{d}{dx}(c_e\theta_e). \tag{3.2}$$

As expansion proceeds, friction between the heavy particles becomes negligible. The rate of ambipolar diffusion is defined only by the acceleration of the charged component, owing to the conversion of thermal electron energy to ion kinetic energy with the aid of an electric field

$$\frac{dv_L}{dx} = -\frac{W}{c_e v_L}\frac{d}{dx}(c_e\theta_e). \tag{3.3}$$

In the given phase of expansion, it is reasonable to assume that energy exchange between electrons and ions does not occur during elastic collisions, nor does recombination take place. The electron temperature θ_e is determined from the electron and ion energy equations

$$\theta_e = H - \frac{0.2}{W} v_L^2, \quad H = c_{e1} v_{L1} x_1^2 \left(\theta_{e1} + \frac{0.2}{W} v_{L1}^2\right). \tag{3.4}$$

The subscript 1 corresponds to a certain fixed cross section r_1. From the electron continuity equation, we have

$$c_e = \frac{N}{x^2 v_L}, \quad N = c_{e1} x_1^2 v_{L1} \quad \text{for} \left(\frac{dn_e}{dt}\right)_p = 0. \tag{3.5}$$

By transforming Eq. (3.3) with the aid of (3.4) and (3.5), we obtain

$$(0.8 v_L^2 - HW)\frac{dv_L}{dy} = (2WHv_L - 0.4 v_L^3)\, y^{-1}, \quad y = \frac{x}{x_1}. \tag{3.6}$$

Integration of (3.6) yields

$$y = \left(\frac{v_L^* + v_L}{v_L^* - v_L}\frac{v_L^* - v_{L1}}{v_L^* + v_{L1}}\right)^{1/v_L^*} \left(\frac{v_L^2}{v_L^{*2} - v_L^2}\frac{v_L^{*2} - v_{L1}^2}{v_{L1}^2}\right)^{-0.25}, \tag{3.7}$$

where $v_L^* = (5HW)^{1/2}$ is the maximum charged-particle velocity.

Figure 5 illustrates the relationship $v_L = v_L(y)$.

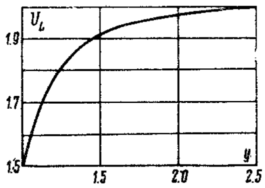

Fig. 5

4. **Variation of the degree of ionization.** Let us examine the variation of the degree of ionization during the expansion of the plasma. The recombination rate is proportional to $n_e^3 T_e^{-9/2}$. Of particular interest is the question of whether recombination is complete or a residual ionization of the gas takes place. We consider a region fairly distant from the source, and for simplicity set $u_L = u_a = \text{const}$.

According to (1.3), the degree of ionization α has the form

$$\frac{d\alpha}{dr} = -\frac{an_m^2}{u_L} T_e^{-9/2}\alpha^3, \qquad \alpha = \frac{n_e}{n_m}. \tag{4.1}$$

The asymptotic behavior of the degree of ionization may be seen from the formal solution of (4.1),

$$\alpha = \alpha_1 \left[1 + \alpha_1^2 2a \int_{r_1}^{r} \frac{n_m^2}{u_L T_e^{9/2}} dr\right]^{-1/2}, \tag{4.2}$$

where the subscript 1 refers to parameters in a certain fixed cross section with radius r_1. If an asymptotic solution for T_e is sought in the form $T_e \sim r^{-p}$, then (assuming $n \sim r^{-2}$ for large r) we find that for $p < 2/3$ the degree of ionization tends to a constant value when $r \to \infty$, while for $p > 2/3$ it tends to zero. The variation of T_e itself, however, depends on the recombination process. For the solution one must explicity determine the relationship $T_e = T_e(r)$.

We examine the phase of sufficiently developed expansion, where the energy exchange between electrons and heavy particles during elastic collisions may be neglected. Changes in electron concentration in this region of the source are defined primarily by the expansion process

$$c_e = c_{e1} y^{-2}. \tag{4.3}$$

With the aid of the substitution $z = \theta_e^{11/2}$, the electron energy equation (1.17) is reduced to the form

$$\frac{dz}{dy} + \frac{22}{3}\frac{z}{y} = \frac{55}{6} R\Pi \frac{c_{e1}^2 x_1}{v_L} y^{-4}. \tag{4.4}$$

By integrating Eq. (4.4), we obtain

$$\theta_e = y^{-4/3}\left[\frac{55}{38} R\Pi c_{e1}^2 \frac{x_1}{v_L}(y^{13/3} - 1) + \theta_e^{11/2}\right]^{2/11}. \tag{4.5}$$

For $y \gg 1$, expression (4.5) can be reduced to a simpler form

$$\theta_e = {}^{55}/_{38} R\Pi c_{e1}^2 y^{-6/11} x_1 v_L^{-1} \text{ or } \quad p = {}^{6}/_{11}. \tag{4.6}$$

Thus, at large distances from the pole, $p < 2/3$ and, consequently, the plasma composition in the supersonic source is frozen.

A zero degree of ionization can be achieved if recombination terminates close to the source pole. This, for example, is possible in the case of very small degrees of ionization.

It should be noted that we have examined only problems associated with the expansion of a plasma in supersonic source. Desirable would be allowance for such important processes as diffusion of radiation, which was not taken into account.

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